NUMBER SYSTEM

Natural Numbers : Numbers which are used for counting the objects are called natural numbers. They are denoted by N.

N = **{ 1, 2, 3……..}**

All positive integers are natural numbers.

Whole numbers :- When 'zero' is included in the natural numbers, they are known as whole numbers.

They are denoted by W.

W= { 0, 1, 2, 3…….}

Integers : All natural numbers, zero and negatives of natural numbers are called as integers.

They are denoted by I.

I = { ……..,-3, -2, **-1, 0, 1 , 2, 3……}**

Rational numbers : The numbers which can be expressed in the form of $\frac{p}{q}$ where P and Q are integers and $q \neq o$ are called rational numbers

They are called by Q.

E.g.= $\frac{1}{2}, \frac{12}{8}, -6$ (as $-6 = \frac{-6}{1}$) etc.

Irrational numbers : The numbers which cannot be written in the form of $\frac{p}{q}$ where P and Q are integers and $q \neq o$ are called irrational numbers.

e.g.- $\sqrt{3}, \sqrt{7}, \frac{2}{17}$ etc

When these numbers are expressed in decimal form, they are neither terminating nor repeating.

e.g.= $\frac{1}{7}, \frac{2}{17}$ etc.

Real numbers : Real numbers include both rational as well as irrational numbers.

Positive or negative, large or small, whole numbers or decimal numbers are all real numbers.

e.g.= 1, 13.79, -0.01, $\frac{2}{3}$ etc.

Imaginary numbers : An imaginary number is a complex number that can be written as a real number multiplied by the imaginary unit **'i'** which is defined by its properly $i^2 = -1$

Note : Zero (0) is considered to be both real and imaginary number.

Prime number : A prime number is a natural number greater than 1 and is divisible only by 1 and itself.

e.g.2, 3, 5, 7, 11, 13, 17 ,19 …….etc.

Note :- 2 is the only even prime number.

Composite Numbers : A number, other than 1, which is not a prime number is called a composite number .

E.g. 4, 6, 8, 9, 10, 12, 14, 15 …….etc.

Note :1 1 is neither a prime number nor a composite number.

2 there are 25 prime numbers between 1 and 100.

To find whether a number is prime or not-

To check whether the number is prime or not,

1 We take an integer larger than the square root of the number. Let the number be **'k'**.

2 Test the divisibility of the given number by every prime number less than **'k'**.

3 If it is not divisible by any of them, then the given number is prime otherwise it is a composite number.

E.g.= Is 881 a prime number ?

Sol- The appropriate square root of 881 is 30.

Prime number less than 30 are 2, 3, 5, 7, 11, 13, 17,19, 23,29.

881 is not divisible by any of the above numbers, so it is a prime minister.

Co-prime numbers : Two numbers are co-prime of their HCF is 1.

E.g. (2,3), (3,4) , (5,7), (3,13) etc.

Even numbers : The number which is divisible by 2 is called even number.

E.g. – 2, 4, **6, 8…….**

Odd numbers – The number which is not divisible by 2 is called odd number.

e.g.= 3, 5, 7, 9……..

Consecutive numbers : A series of numbers in which the succeeding number is greater then the preceding number by 1 is called a series of consecutive numbers.

i.e., Difference between two consecutive numbers is 1.

Some Rules on Counting Numbers

1. Sum of all the first n natural numbers

$$= \frac{n(n+1)}{2}$$

Q. Find the sum of first 20 natural numbers.

Ans- Sum of 1 to 20

$$\text{Sum of 1 to 20}$$
$$\frac{20(20+1)}{2} = 210$$

Q. Find the sum of numbers from 11 to 20.

Ans Sum of 1 to 20 = $\frac{20(20+1)}{2} = 210$

Sum of 1 to 10 = $\frac{10(10+1)}{2} = 55$

Sum of 11 to 20= 210-55=155

2. Sum of first n old numbers =

$$n^2$$

Q. What is the sum of first 10 odd numbers ?

Ans- Sum of first 10 odd numbers = $(10)^2 = 100$

Q. Find the sum of 9+11+13+……+29

Ans – **1+3+5+……+29=** $(15)^2 = 225$

(as there are 15 odd numbers from 1 to 29)

1+3+5+7= $(4)^2 = 16$

9+11+13+29=225-16=209

3. Sum of first n even numbers

$$n(n+1)$$

Q. What is the sum of even numbers between 1 and 50 ?

Ans – No. of even numbers between 1 and 50 = $\frac{50}{2} = 25$

Sum of even numbers between 1 and 50

$= 25(25+1) = 25 \times 26 = 650$

Q. Find the value of 12+14+…….+30.

Ans- **(2+4+6+…….+30) has 15 even numbers**

2+4+6+……30=15(15+1)=240

Similarly 2+4+6+8+10= 5(5+1)=30

12+14…….+30=240-30=210

4. Sum of squares of first n natural numbers

$$=\frac{n(n+1)\ (2n+1)}{6}$$

Q. what is the value of $1^2+2^2+.....+10^2$?

Ans- $1^2+2^2+.....+10^2$?

$$=\frac{10(10+1)\ (2\times10+1)}{6}$$

$$=\frac{10\times11\times21}{6}=385$$

5. Sum of cubes of first n natural numbers.

$$=\left[\frac{n(n+1)}{2}\right]^2$$

Q. What is the value of $1^3+2^3+.....+5^3$?

Ans- $1^3+2^3+.....+5^3$

$$=\left[\frac{5(5+1)}{2}\right]^2=\left[\frac{5\times6}{2}\right]^2=225$$

Important Formulas of Number System

Formulas of Number Series

1. $1 + 2 + 3 + 4 + 5 + ... + n = n(n + 1)/2$
2. $(1^2 + 2^2 + 3^2 + ... + n^2) = n (n + 1) (2n + 1) / 6$
3. $(1^3 + 2^3 + 3^3 + ... + n^3) = (n(n + 1)/ 2)^2$
4. Sum of first n odd numbers = n^2
5. Sum of first n even numbers = n (n + 1)

Mathematical Formulas

1. $(a + b)(a - b) = (a^2 - b^2)$
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$
9. $(a + b)^n = a^n + ({}^nC_1)a^{n-1}b + ({}^nC_2)a^{n-2}b^2 + ... + ({}^nC_{n-1})ab^{n-1} + b^n$

DIVISIBILITY RULES

Divisible by 2 – If a number ends with 0,2,4,6,8 then the number is divisible by 2.

Example – 254, 326, 3548, 4210. All number ends with 4,6,8,0 so these numbers are divisible by 2.

Divisible by 3 – If sum of all the digits of a number is divisible by 3, then the number itself, is also divisible by 3.

Example – 375,4251,78123. Here we are taking another example of 549 – 5+4+9 = 18 which is divisible by 3, so 549 is also divisible by 3.

Divisible by 4 – If the last two digits of any number is divisible by 4, then the number is also divisible by 4.

Example – 2348.Here last two digits 48 are divisible by 4, so 2348 is also divisible by 4.

Divisible by 5 – If a number ends with 0 or 5, then it is divisible by 5.

Example – 340, 625.

Divisible by 6 – If a number is divisible by both 2 and 3, then it is divisible by 6 as well.

Example – 4536. Here the number ends on 6 which is divisible by 2 and the sum of digits (4+5+3+6 = 18) which is divisible by 3 also, so the number 4536 is divisible by 6.

Divisible by 8 – If the last three digits of a number can be divided by 8, then the number is divisible by 8.

Example – 746848. Here last 3 digits 848 are divided by 8, hence the number 746848 is divisible by 8.

Divisible by 10 – If a number ends with 0, then it is divisible by 10.

Example – 120, 330, 500.

Divisible by 11 – If (sum of its digit in odd places) is subtracted by (sum of its digits in even places) = 0 or multiple of 11, then the number is divisible by 11.

Example – 39798846Sum of digits at odd places = 3+7+8+4 = 22 Sum of digits at even places = 9+9+9+6 = 33Now -> 33-22 = 11 which is multiple of 11 so number is divisible.

Divisible by 12 – If a number is divisible by 3 and 4 both, then it will also be divisible by 12 as well.

Example – 4848 is divisible by 3 and 4 both, so it will be divisible by 12 also.

Divisible by 14 – If a number is divisible by 2 and 7 both, then it will also be divisible by 14 as well.

Example – 4242 is divisible by 2 and 7 both, so it will be divisible by 14 also.

Divisible by 15 – If a number is divisible by 3 and 5 both, then it will also be divisible by 15 as well.

Example – 4545 is divisible by 3 and 5 both, so it will be divisible by 15 also.

Divisible by 16 – A number is divisible by 16, if the number formed by the last4 digits is divisible by 16.

Example – 7957536 Last four digits 7536 are divisible by 16.

Divisible by 24 – If a number is divisible by 3 and 8 both, then it will also be divisible by 14 as well.

Example – 4848 is divisible by 3 and 8 both, so it will be divisible by 24 also.

Divisible by 40 – If a number is divisible by 5 and 8 both, then it will also be divisible by 40 as well.

Example – 8080 is divisible by 5 and 8 both, so it will be divisible by 40 also.

Divisible by 80 – If a number is divisible by 5 and 16 both, then it will also be divisible by 80 as well.

Example – 80160 is divisible by 5 and 16 both, so it will be divisible by 80 also.

QUESTIONS FOR PRACTICE

Which one of the following is not a prime number?

A. 31

B. 61

C. 71

D. 91

Answer: Option **D**
Explanation:
91 is divisible by 7. So, it is not a prime number.

$(112 \times 5^4) = ?$

A. 67000

B. 70000

C. 76500

D. 77200

Answer: Option **B**
Explanation:

$(112 \times 5^4) = 112 \times \left(\frac{10}{}\right)^4 = \frac{112 \times 10^4}{} = \frac{1120000}{} = 70000$

It is being given that $(2^{32} + 1)$ is completely divisible by a whole number. Which of the following numbers is completely divisible by this number?

A. $(2^{16} + 1)$

B. $(2^{16} - 1)$

C. (7×2^{23})

D. $(2^{96} + 1)$

Answer: Option **D**
Explanation:
Let $2^{32} = x$. Then, $(2^{32} + 1) = (x + 1)$.
Let $(x + 1)$ be completely divisible by the natural number N.
Then,
$(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$, which is completely divisible by N, since $(x + 1)$ is divisible by N.

What least number must be added to 1056, so that the sum is completely divisible by 23 ?

A. 2

B. 3

C. 18

D. 21

E. None of these

Answer: Option **A**
Explanation:

```
23) 1056 (45
      92
      ---
      136
      115
      ---
       21
      ---

Required number = (23 - 21)
                = 2.
```

1397 x 1397 = ?

A. 1951609

B. 1981709

C. 18362619

D. 2031719

E. None of these

Answer: Option **A**
Explanation:
$1397 \times 1397 = (1397)^2$

$= (1400 - 3)^2$

$= (1400)^2 + (3)^2 - (2 \times 1400 \times 3)$

$= 1960000 + 9 - 8400$

$= 1960009 - 8400$

$= 1951609.$

How many of the following numbers are divisible by 132 ?
264, 396, 462, 792, 968, 2178, 5184, 6336

A. 4

B. 5

C. 6

D. 7

Answer: Option **A**
Explanation:

132 = 4 x 3 x 11

So, if the number divisible by all the three number 4, 3 and 11, then the number is divisible by 132 also.

264 → 11,3,4 (/)

396 → 11,3,4 (/)

462 → 11,3 (X)

792 → 11,3,4 (/)

968 → 11,4 (X)

2178 → 11,3 (X)

5184 → 3,4 (X)

6336 → 11,3,4 (/)

Therefore the following numbers are divisible by 132 : 264, 396, 792 and 6336.

Required number of number = 4.

$(935421 \times 625) = ?$

A. 575648125

B. 584638125

C. 584649125

D. 585628125

Answer: Option **B**
Explanation:

$935421 \times 625 = 935421 \times 5^4 = 935421 \times \left(\frac{10}{2}\right)^4$

$= \frac{935421 \times 10^4}{2^4} = \frac{9354210000}{16}$

$= 584638125$

Q. The largest 4 digit number exactly divisible by 88 is:

A. 9944

B. 9768

C. 9988

D. 8888

E. None of these

Answer: Option **A**
Explanation:

```
Largest 4-digit number = 9999

 88) 9999 (113
     88
     ----
     119
      88
     ----
      319
      264
      ---
       55
      ---

 Required number = (9999 - 55)
                 = 9944.
```

Q. Which of the following is a prime number ?

A. 33

B. 81

C. 93

D. 97

Answer: Option **D**
Explanation:
Clearly, 97 is a prime number.

Q. What is the unit digit in $\{(6374)^{1793} \times (625)^{317} \times (341^{491})\}$?

A. 0

B. 2

C. 3

D. 5

Answer: Option **A**
Explanation:
Unit digit in $(6374)^{1793}$ = Unit digit in $(4)^{1793}$
= Unit digit in $[(4^2)^{896} \times 4]$

= Unit digit in $(6 \times 4) = 4$

Unit digit in $(625)^{317}$ = Unit digit in $(5)^{317} = 5$
Unit digit in $(341)^{491}$ = Unit digit in $(1)^{491} = 1$

Required digit = Unit digit in $(4 \times 5 \times 1) = 0$.

Q. 5358 x 51 = ?

A. 273258

B. 273268

C. 273348

D. 273358

Answer: Option **A**
Explanation:
5358 x 51 = 5358 x (50 + 1)

= 5358 x 50 + 5358 x 1

= 267900 + 5358

= 273258

Q. The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number ?

A. 240

B. 270

C. 295

D. 360

Answer: Option **B**
Explanation:
Let the smaller number be x. Then larger number = $(x + 1365)$.
$\therefore$ $x + 1365 = 6x + 15$
$\Rightarrow$ $5x = 1350$
$\Rightarrow$ $x = 270$

$\therefore$ Smaller number = 270.

Q. $(12)^3 \times 6^4 \div 432 = ?$

A. 5184

B. 5060

C. 5148

D. 5084

E. None of these

Answer: Option **A**
Explanation:
Given Exp. $= \frac{(12)^3 \times 6^4}{432} = \frac{(12)^3 \times 6^4}{12 \times 6^2} = (12)^2 \times 6^2 = (72)^2 = 5184$

Q. 72519 x 9999 = ?

A. 725117481

B. 674217481

C. 685126481

D. 696217481

E. None of these

Answer: Option **A**
Explanation:
72519 x 9999 = 72519 x (10000 - 1)

= 72519 x 10000 - 72519 x 1

= 725190000 - 72519

= 725117481.

Q. If the number 517*324 is completely divisible by 3, then the smallest whole number in the place of * will be:

A. 0

B. 1

C. 2

D. None of these

Answer: Option **C**

Explanation:
Sum of digits = (5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x), which must be divisible by 3.
$\therefore$ x = 2.

Q. The smallest 3 digit prime number is:

A. 101

B. 103

C. 109

D. 113

Answer: Option **A**
Explanation:

The smallest 3-digit number is 100, which is divisible by 2.

$\therefore$ 100 is not a prime number.

101 < 11 and 101 is not divisible by any of the prime numbers 2, 3, 5, 7, 11.

$\therefore$ 101 is a prime number.

Hence 101 is the smallest 3-digit prime number.

Q. Which one of the following numbers is exactly divisible by 11?

A. 235641

B. 245642

C. 315624

D. 415624

Answer: Option **D**
Explanation:

(4 + 5 + 2) - (1 + 6 + 3) = 1, not divisible by 11.

(2 + 6 + 4) - (4 + 5 + 2) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 3) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 4) = 0, So, 415624 is divisible by 11.

Q. (?) - 19657 - 33994 = 9999

A. 63650

B. 53760

C. 59640

D. 61560

E. None of these

Answer: Option **A**
Explanation:

```
 19657          Let x - 53651  =
9999
 33994          Then, x = 9999 +
53651 = 63650
 -----
 53651
 -----
```

Q. The sum of first 45 natural numbers is:

A. 1035

B. 1280

C. 2070

D. 2140

Answer: Option **A**
Explanation:
Let S_n =(1 + 2 + 3 + ... + 45). This is an A.P. in which a =1, d =1, n = 45.

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{45}{2} \times [2 \times 1 + (45-1) \times 1] = \left(\frac{45}{2} \times 46\right) = (45 \times 23)$

= 45 x (20 + 3)

= 45 x 20 + 45 x 3

= 900 + 135

= 1035.

Shorcut Method:

$S_n = \frac{n(n+1)}{2} = \frac{45(45+1)}{2} = 1035.$

Q. Which of the following number is divisible by 24 ?

A. 35718

B. 63810

C. 537804

D. 3125736

Answer: Option **D**
Explanation:

24 = 3 x8, where 3 and 8 co-prime.

Clearly, 35718 is not divisible by 8, as 718 is not divisible by 8.

Similarly, 63810 is not divisible by 8 and 537804 is not divisible by 8.

Consider option (D),

Sum of digits = (3 + 1 + 2 + 5 + 7 + 3 + 6) = 27, which is divisible by 3.

Also, 736 is divisible by 8.

$\therefore$ 3125736 is divisible by (3 x 8), *i.e.,* 24.

Q. $\frac{753 \times 753 + 247 \times 247 - 753 \times 247}{753 \times 753 \times 753 + 247 \times 247 \times 247}$ **= ?**

A. $\frac{1}{1000}$

B. $\frac{1}{506}$

C. $\frac{253}{500}$

D. None of these

Answer: Option **A**
Explanation:
Given Exp. $= \frac{(a^2 + b^2 - ab)}{(a^3 + b^3)} = \frac{1}{(a+b)} = \frac{1}{(753 + 247)} = \frac{1}{1000}$

Q. (?) + 3699 + 1985 - 2047 = 31111

A. 34748

B. 27474

C. 30154

D. 27574

E. None of these

Answer: Option **B**
Explanation:
x + 3699 + 1985 - 2047 = 31111
$\Rightarrow$ x + 3699 + 1985 = 31111 + 2047
$\Rightarrow$ x + 5684 = 33158
$\Rightarrow$ x = 33158 - 5684 = 27474.

Q. If the number 481 * 673 is completely divisible by 9, then the smallest whole number in place of * will be:

A. 2

B. 5

C. 6

D. 7

E. None of these

Answer: Option **D**
Explanation:
Sum of digits = (4 + 8 + 1 + x + 6 + 7 + 3) = (29 + x), which must be divisible by 9.
$\therefore$ x = 7.

Q. The difference between the local value and the face value of 7 in the numeral 32675149 is

A. 75142

B. 64851

C. 5149

D. 69993

E. None of these

Answer: Option **D**
Explanation:
(Local value of 7) - (Face value of 7) = (70000 - 7) = 69993

Q. The difference between a positive proper fraction and its reciprocal is 9/20. The fraction is:

A. $\frac{3}{5}$

B. $\frac{3}{10}$

C. $\frac{4}{5}$

D. $\frac{4}{3}$

Answer: Option **C**
Explanation:

Let the required fraction be x. Then $\frac{1}{x} - x = \frac{9}{20}$

$\therefore$ $\frac{1 - x^2}{x} = \frac{9}{20}$

$\Rightarrow$ $20 - 20x^2 = 9x$
$\Rightarrow$ $20x^2 + 9x - 20 = 0$
$\Rightarrow$ $20x^2 + 25x - 16x - 20 = 0$
$\Rightarrow$ $5x(4x + 5) - 4(4x + 5) = 0$
$\Rightarrow$ $(4x + 5)(5x - 4) = 0$
x = $\frac{4}{5}$

Q. On dividing a number by 56, we get 29 as remainder. On dividing the same number by 8, what will be the remainder ?

A. 4

B. 5

C. 6

D. 7

Answer: Option **B**
Explanation:

Formula: (Divisor*Quotient) + Remainder = Dividend.

Soln:

(56*Q)+29 = D -------(1)

D%8 = R ----------(2)

From equation(2),

((56*Q)+29)%8 = R.

=> Assume Q = 1.

=> (56+29)%8 = R.

=> 85%8 = R

=> 5 = R.

Q. If n is a natural number, then ($6n^2 + 6n$) is always divisible by:

A. 6 only

B. 6 and 12 both

C. 12 only

D. by 18 only

Answer: Option **B**
Explanation:
($6n^2 + 6n$) = $6n(n + 1)$, which is always divisible by 6 and 12 both, since $n(n + 1)$ is always even.

Q. 107 x 107 + 93 x 93 = ?

A. 19578

B. 19418

C. 20098

D. 21908

E. None of these

Answer: Option **C**
Explanation:

107 x 107 + 93 x 93 $= (107)^2 + (93)^2$

$= (100 + 7)^2 + (100 - 7)^2$

$= 2 \times [(100)^2 + 7^2]$ [**Ref:** $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$]

$= 20098$

Q. What will be remainder when (67^{67} + 67) is divided by 68 ?

A. 1

B. 63

C. 66

D. 67

Answer: Option **C**
Explanation:
(x^n + 1) will be divisible by (x + 1) only when n is odd.
$\therefore$ (67^{67} + 1) will be divisible by (67 + 1)
$\therefore$ (67^{67} + 1) + 66, when divided by 68 will give 66 as remainder.

Question 1: The least number which when divided by 6, 9, 12, 15 and 18 leaves the same remainder 2 in each case is :
a) 180
b) 182
c) 178
d) 176

Question 2: What is the arithmetic mean of first 20 odd natural numbers ?
a) 19
b) 17
c) 22
d) 20

Question 3: The least number that should be added to 2055, so that the sum is exactly divisible by 27 is
a) 28
b) 24
c) 27
d) 31

Question 4: The digit in the unit place in the square root of 66049 is
a) 3
b) 7
c) 8
d) 2

Question 5: A certain sum will amount to 12,100 in 2 years at 10% per annum of compound interest, interest being compounded annually. The sum is
a) 8000
b) 6000
c) 12000
d) 10000

SOLUTIONS

1) Answer (b)
The numbers 6,9,12,15,18 leaves same remainder 2 in each case. So, what we need to do is find the L.C.M. of these numbers and add 2 to it L.C.M. of 6,9,12,15,18 = 180 =>Required no. = 180+2 = 182

2) Answer (d)
NOTE :- Sum of first 'n' odd natural numbers = n^{2} Sum of first 'n' even natural numbers = n(n+1) Sum of first 20 odd natural numbers = 20^{2} = 400
Arithmetic mean = 400/20 = 20

3) Answer (b)
The remainder obtained by dividing 2055 by 27 = 3
So, the least number that should be 'subtracted' from 2055 to make it perfectly divisible by 27 = 3 and the least number that should be added = 27-3 = 24

4) Answer (b)
Square root of 66049 = 257 Thus, unit's digit = 7

5) Answer (d)